NP-COMPLETENESS HOMEWORK

Problem 1

The Traveling Salesman Problem is a classic NP-Hard problem where, given an edge-weighted graph $G = (V, E)$, the goal is to travel in a cycle to each node in V so as to minimize the total sum of weights on the cycle taken. Consider the NP-Complete decision version of this problem in the below.

For each of the questions below, decide whether the answer is i) Yes, ii) No, or iii) Unknown, because it would resolve the question of whether P = NP. Give a brief explanation of your answer.

a) Does the Shortest Path Problem $\leq_p$ The Traveling Salesman Problem?
b) Does the Traveling Salesman Problem $\leq_p$ the Shortest Path Problem?

Problem 2

The Graph Isomorphism problem is the problem of determining if a mapping f can be created between graphs $G = (V, E)$ and $H = (W, F)$ such that $f(v) = w$ for $v \in V$ and $w \in W$ and so that for any $(u, v) \in V$, $(f(u), f(v))$ is an edge in F if and only if (u, v) is an edge in E.

Prove that the Graph Isomorphism Problem is in NP. In order to prove that a problem is in NP, recall that you need to create a polynomial-time certifier. Thus, your proof consists of the following parts:

a) A description of the certifier algorithm.
b) Pseudocode for the certifier.
c) A time analysis showing that the certifier is polynomial-time.

Problem 3

Prove that the Registrar's Problem is NP-Complete. Consider the decision version of the basic registrar's problem: is it possible to create a schedule with a given student preferences value?