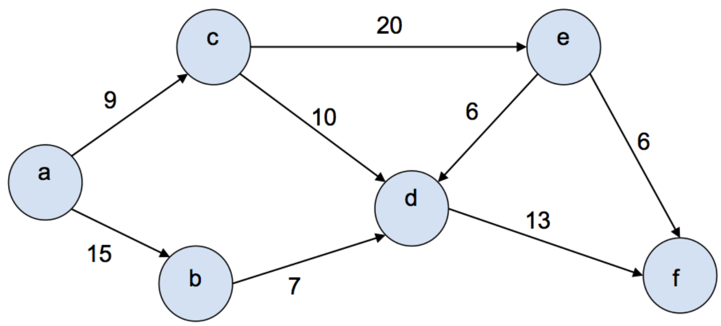
NETWORK FLOW HOMEWORK

Problem 1

Run the Ford-Fulkerson algorithm on the graph below with source a and sink f. Draw the graph as well as the residual graph after each augmenting path is chosen. For each iteration, pick the augmenting path that is lexicographically smallest (using the node letters).



Problem 2

Decide whether each of the following statements is true or false. If it is true, give a short explanation. If it is false, give a counterexample. For all of these statements, let $G = (V, E)$ be an arbitrary flow network, with a source s, a sink t, and a positive integer capacity $c(e)$ on every edge e. Let (A, B) be a minimum s-t cut with respect to these capacities $c(e)$ for e in E.

i) Suppose we multiply each $c(e)$ by 2. Then (A, B) is still a minimum s-t cut with respect to these new capacities $c'(e) = 2 \cdot c(e)$.

ii) Suppose we subtract 1 from each $c(e)$ (assuming the previous minimum $c(e)$ value was 1). Then (A, B) is still a minimum s-t cut with respect to these new capacities $c'(e) = c(e) - 1$.

iii) Suppose we square each $c(e)$ value. Then (A, B) is still a minimum s-t cut with respect to these new capacities $c'(e) = c(e) \cdot c(e)$.

Problem 3 - full algorithm design write-up

In preparation for Black Friday, consider the following problem. There are k stores in the region that sell the latest must-have kids toy. There are n parents in the area who want the toy, and they decide to cooperate to make sure they each get one without having to drive too far. They decide they each only want to drive for at most 1 hour from their house. The stores, also in a cooperating mood, agree to each order $\lceil n/k \rceil$ (rounded up) toys. Give an algorithm to determine whether it's possible for each parent to get a toy without driving for more than an hour.